**On the Existence, Stability Analysis and Optimal Control of a Mathematical Model to****Attenuate the Spread of Head Lice Infestation.**

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***Abstract***

In spite of effort by public health officials to attenuate head lice, it remains endemic in several part of the globe both in developing and developed countries. This work presents a deterministic mathematical model and optimal control to attenuate head lice infection. We validate the proposed model by studying it existence and uniqueness solution, computed the basic reproduction number  and the local stability analysis associated with the disease-free equilibrium point. In addition, we extended the proposed model to have optimal control model, analysed it using the most celebrated Pontryagins Maximum Principle and solved numerically by means of forward-backward sweep fourth-order Runge-Kutta approach in Octave/MATLAB. Furthermore, we performed numerical simulation to see the effect of the optimal control on the infected, treated and recovered population. Sequel to the simulation, we observed that better result is achieved with the optimal control model compared to the proposed model without optimal control as the number of the infected and treated individuals decrease and increase respectively, with increase in the recovered individuals within a stipulated time interval.

***Keywords***

Pediculosis, Stability Analysis, Existence Solution, Uniqueness Solution, Basic Reproduction Number, Mathematical Model and Optimal Control.

1. **Introduction**

Generally speaking, pediculus humanus capitis normally referred to as head louse is not a disease that threatens human life, however it has remained a source of disturbance to siblings, parents, schools of an infested children and public health officials for several decades. The state of being infested with head lice is called pediculosis capitis. Head lice often leave close to the scalp of the head where they extremely feeds on human blood as source of diet and maintain its body temperature. Pediculus humanus capitis also referred to as head lice among other lice such as Phthirus pubis commonly referred to as crab or pubic lice and pediculus humanus corpon’s commonly called clothing or body lice are the only lice out of thousands of ectoparasite in existence that infest human being [1].

There are three stages head lice go through in their life cycle and they are egg, nymph and adult stage. The eggs (nits) are lay by adult female louse. It takes eight to nine days period to hatch and are often located at the base of the hair shaft where they are usually mistaken for dandruff or hair spray droplets due to their tiny nature [2]. After hatching, it takes seven to twelve days for the nymph to become adult and it feeds extremely on human blood as it main source of diet and survival. In this stage, the nymph looks like an adult head louse but it has a pinhead size [2]. The last and not the least stage, is the adult stage. In this stage, the features of adult louse are seen such as legs, actual size and colour. It has six legs, tan to grayish white in colour and its size is about the size of a sesame seed. The survival period for the adult louse on human head where it lives and feeds on human blood as it main source of diet three to ten (3-10) times per day is thirty (30) days [3].

The major places of contacting head lice are in kindergartens, primary and junior secondary schools where head-to-head contact is likely to happen during classroom activities, during play and during school bus riding; thus transferring same disease to their family members at home via head-to-head contact [4]. This is not far from truth because head nice neither crawl nor hop or fly therefore having head-to-head contact as it only means of contact with an infected individual [5]. Head lice symptoms are tickling feeling of moving object in the hair, itching which often take place in four to six (4-6) weeks period (not excluding first day of infection) after human host develops allergic reaction to head lice saliva, and scratching. There are different kinds of treatment options for pediculosis which are considered effective if the victim is “head lice-free” after receiving the last treatment dose and staying fourteen days without showing signs of head lice [6]. These treatment options are oral treatment, classical topical use of pediculicide such as pyrethis and malathion, and therapeutic wet combing and conditioner, which when used sequel to early detection of head lice infestation in the host population will help put an end to the disease and its spread [7].

In recent years, several mathematical models to study and analyse the dynamics of ectoparasites such as ticks [8], flees [9] and sea lice [10] have emerged. In [11], the authors presented a deterministic stochastic model to study the dynamics of head lice based on stochastic susceptible – infectious – susceptible (SIS). Their analysis shows that if the basic reproduction number is greater than zero, it will take long time for the disease to go on extinction. In addition, [12] presented an endemic infectious model to study the dynamics of head lice infestation. The authors used data collected from recent study in Welsh schools on the prevalence of head lice infestation to gain insight on the dynamics of head lice in a host population using the stationary distribution of the number of infected individuals. The model parameters were obtained using maximum likelihood methods. Their work reveals that if the probability is one, the disease will go on extinction and there wouldn’t be stationary distribution.

In 1956 a Russian Mathematician by name Lev Pontryagin and his associates formulated the most “celebrated pontryagin maximum principle” use in optimal control theory to find out the best control strategies needed for taking a dynamical system from one form to another [13]. “Roughly speaking, Pontrayagin Maximum Principle (PMP) states that it is necessary for any optimal control along with the optimal state trajectory to satisfy the so called Hamiltonian system, plus a maximality condition on the Hamiltonian” [14]. Diverse real life scenarios have attracted extremely the application of optimal control theory. Take for instance, it has been applied in evolving effective control strategies for life threatening disease and mitigating the spread of deadly diseases such as HIV/AIDS [14], cancer [15] and swine flu [16].

Our proposed model is an extension of [17] where the authors have proposed and analysed a continuous time deterministic mathematical model for head lice infestation based on susceptible – infectious – recovered (SIR). In their work, total population was considered fixed and the disease transmission follows a linear standard incidence. Hence we formulated a mathematical model for head lice infection by incorporating a new compartment called treated compartment denoted as T(t). Later we applied optimal control theory to minimize the spread of head lice in a host population and to get the optimal control profile relating to transmission rate and treatment rate using two control strategies namely education and treatment.

The remaining sections of this work are organized as follows: In section two, we described the proposed model, its parameters, existence and uniqueness of solution, basic properties of the model, model analysis and discussion of numerical simulation and results. In section three, we discuss the optimal control model and its analysis. In section four, we describe the numerical simulation results of the optimal control model. Lastly, in section five, we conclude the work.

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1. **Mathematical Model Formulation, Analysis and Numerical Simulation.**

**2.1 Parameters and State Variable Description**

**Table 1.** Model Parameter and their description

|  |  |  |
| --- | --- | --- |
| **Parameter** | **Description** |  |
|  | Recruitment rate (immigration). |  |
|  |  |  |
|  | Head lice transmission rate |  |
|  |  |  |
|  | Rate at which recovered individual move to susceptible class. |  |
|  |  |  |

 Rate of recovery from the disease

 Treatment rate

 Natural death rate.

**Table 2.** Model State Variable and their description

|  |  |  |
| --- | --- | --- |
| **State Variable** | **Description** |  |
| Susceptible (S) | The class of individuals susceptible at time t. |  |
|  |  |  |
| Infectious (I) | The class of individual infested at time t. |  |
|  |  |  |
| Treated (T) | The class of individuals who are treated at time t. |  |
|  |  |  |

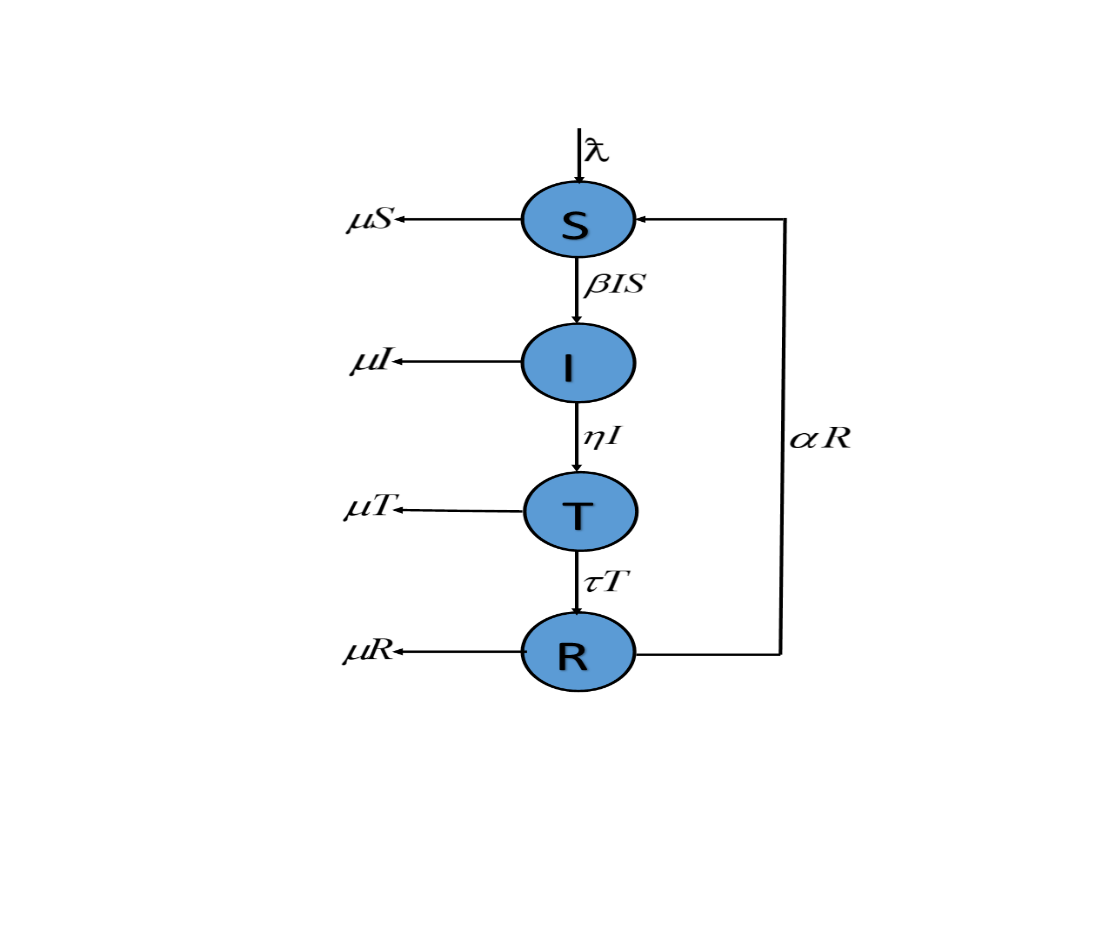
Recovered (R)The class of individuals who recovered

at time t.

Population (N) The total population at time t.

**2.2 Model Flow Diagram**

The compartmental flow diagram below provides a summary insight on the description of head lice disease dynamics in a host population.

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**Fig. 1** Model flow diagram.

**2.3 Model Formulation**

In this research work, we extend the model proposed in [17] by incorporating a new compartment called treatment denoted as T(t). The model divide the host population N(t) of a real life system into four compartments namely Susceptible Compartment S(t), Infected Compartment I(t), Treatment Compartment T(t) and Recovered Compartment R(t); where the compartment S(t) refers to individuals who are likely to be infected by the disease, the compartment I(t) refers to individuals who are infected and are able to infect others, the compartment T(t) refers to individuals who are treated of the disease and the compartment R(t) refers to those individuals who have recovered from the disease with temporary immunity. We formulate a mathematical model governing the dynamics of pediculosis spread in a real life system by making use of the assumptions below:

1. The host population N(t) is varying and mix homogeneously, meaning that every individuals in the population are likely to be infested.
2. There is constant influx into the population at the rate  because of migration.
3. The non-life threatening disease is transferred via head-to-head contact between susceptible individuals and infested individuals at the linear incidence rate  where  is the infection rate and
4. There is no death due to the disease, however there is natural death at the rate for individuals in all compartment and 
5. The infested individuals are treated at the rate  and recovered without permanent immunity.

With the model flow diagram in subsection (2.2) and the assumption in subsection (2.3) in mind, we propose a mathematical model for dynamics of pediculosis as follows:



Where is incidence rate due to migration,is the rate at which recovered individuals return to susceptible compartment due to temporary immunity to the disease, is the infection rate, is the natural death rate,  is the rate at which infested individuals move into the treated compartment, is the rate at which treated individuals recover after treatment and move into the recovered compartment.

Thus, the total population is given as.



**2.4 Existence and Uniqueness solution of the Formulated Model**

Most Real-life problems are being governed by mathematical models [18]. The authenticity and validity of these models are being determined by the existence and uniqueness of the model solution. In this section, we formulate theorem on existence of unique solution of the system in equation (1) – (4) above and provide the proof.

Let the system of model equation (2.1) – (2.4) be



Also considering the system of equations below





Equation (2.7) above is the compact form of equation (2.6).

**Theorem 1:** [19]

Let the region of consideration be , then



Suppose that satisfies the Lipchitz condition then



where k is a positive constant, whenever the pairs  and belong to . Then, there is a constant  such that there exist a unique continuous vector solution of the system (2.5) in the interval  It is sacronsact to note that condition (2.8) is satisfied by the requirement that  are continuous and bounded in  with respect to equation (2.5).

The region of interest is



and a bounded solution of the form



is found in the region , whose partial derivatives satisfies where and are positive constants.

**Theorem 2**: Let the region defined in (2.9) be denoted by  such that equation (2.9) - (2.10) hold. Then, the system of equation (2.5) has a unique solution which is continuous and bounded in the region.

**Proof**

Recall equation (5)



We show that where  are continuous.

For G1, we have

,,  and .

For G2, we have

   and 

For G3, we have

   and 

For G4, we have

   and 

From the computation above, it is obvious all the partial derivate exist, continuous and bounded. Thus, theorem 2 above shows that there exist unique solution of system of equation (2.5) in the region.

**2.5 Basic Properties of the Model**

In this subsection, we study some basic properties of the system of equation in (2.1) – (2.4), namely positivity and boundedness of solution and region of feasible solution. The positivity and boundedness of solution shows the positivity of the model equations while the region of feasible solution is the region in which the solution to the model equation (2.1) – (2.4) make sense biologically.

**2.5.1 Region of Feasible Solution**

Let  be the feasible solution set which is positively invariant set of the model.

**Lemma 1.** The feasible solution set is positive and attract all solution in.

**Proof**

Since the host population is the sum of the four compartments and, that is



Thus









Solving the first order linear differential equation above we have



When the equation above yields



When the equation above becomes 

Thus, the set of feasible solution is positively invariant and attracts all solution in 

**2.5.2 Positivity and Boundedness of Solution**

We have shown that the solution set to the model equation is feasible and attracts all solution in  we now show that the variables of the model equation (2.1) – (2.4) are non-negative.

**Lemma2.** Let be initial state variable values, then the set of solutions  of equation (1) – (4) is positive for all 

**Proof** Let’s consider the system of equation (1)-(4).

From equation (1), if we assume that



Solving the equation above for S(t) we have



When we have



Therefore, 

In like manner, from equation (2),



Solving the equation above for I(t), we have



When  we have



Therefore, 

Also, from equation (3),



Solving the equation above for T(t), yields



When  we have



Therefore, 

Lastly, from equation (4)



Solving the equation above for R(t), yields



When  we have



Therefore, 

Thus, we have established that all state variables are positive 

**2.6 Model Analysis**

This section analysed qualitatively the system of model equation (2.1) – (2.4) to investigate the equilibrium point and their respective stability analysis to understand the dynamics of pediculosis in a host population.

**2.6.1 Disease Free Equilibrium (DFE) Point**

Let the disease-free equilibrium (DFE) point be denoted by We obtain the disease free equilibrium point of the system of model equation (2.1) – (2.4) by setting



Equation (2.1) – (2.4) becomes



In absence of infection (that is I = 0), equation (2.11) yields



Solving equation (2.13) for T, we have



In like manner, solving equation (2.14) for R, yields



Lastly, solving equation (2.12) for S, yields



Therefore, the disease-free equilibrium (DFE) point of the system of model equation (2.1) - (2.4) denoted by  exist and is express as



**2.6.2 Reproduction Number (Ro)**

The basic reproduction number denoted byis defined as the number of secondary infection produced by a single infectious individual when introduced into a completely susceptible population [20]. The persistence or die out of a disease in a population solely depends on the value of the computed reproduction number. If the value ois less than zero (< 0), implies that the disease will die out, leaving the infectious individuals with no choice but to produce less than one secondary infection. However, if the value of greater than zero (> 0) implies that the disease will persist in the population, thus producing more than one secondary infection and resulting to epidemic

In most complex epidemic models, basic reproduction number is usually computed using the next generation approach. Nevertheless, since our proposed model has only one infectious compartment, we adopt and use the approach in [21] for computing basic reproduction number.

Let



Where

 : Constant of proportionality (=1)

 : The probability of infection, given contact between susceptible and infected.

 : The rate in which the susceptible individual have contact with infected individual.

: The duration of infectiousness.

From equation (2.2) of the system of model equations above,



are rate of removal from the infection compartment. Thus,  becomes the rate of removal from the infectious compartment



Almost every individual in a given population is susceptible beside the case where there is index at the beginning of any epidemic [21]. Thus, susceptible individual (S) is approximately unity that is 



At t = 0, we have that



From the equation above, as , decreases.

If the number of infectious individual increases, then epidemic occurs and otherwise 

Thus, equation (2.2) above becomes less than zero,



Recall from disease free equilibrium point that 

Thus, 

**2.6.3 Local Stability of Disease-Free Equilibrium (DFE) point**

To obtain the local stability analysis of disease-free equilibrium point, we calculate the Jacobian matrix and evaluate it at the disease free equilibrium point.

Let the system of model equation (1) - (4) be expressed as



The Jacobian matrix is defined as



At the disease free equilibrium point, equation (2.15) becomes



In other to obtain the local stability of the disease-free equilibrium point, we adopt and use the Routh-Hurwitz condition which state that



With respect to the Jacobian matrix in equation (2.16) above.

If the condition in equation (2.18) is satisfied, then the disease-free equilibrium point is locally asymptotically stable otherwise it is locally asymptotically unstable.

From equation (2.16), we have that



Also, from equation (17), we have that



Recall that



Thus,



Since condition (18) above is satisfied, hence the disease-free equilibrium point  is asymptotically stable provided  thus the disease does not persist (it dies out) within a time interval.

**2.7 Numerical Simulation**

In this section, we performed a numerical simulation to give more insights to our analytical results. Some of the parameters used are taken from [17] and the remaining parameters were estimated and assumed. Using MATLAB, we performed the numerical simulation of the system of model equation (2.1) – (2.4) subject to the initial conditions given as and different set of parameter values as are captured in the table below at final time 

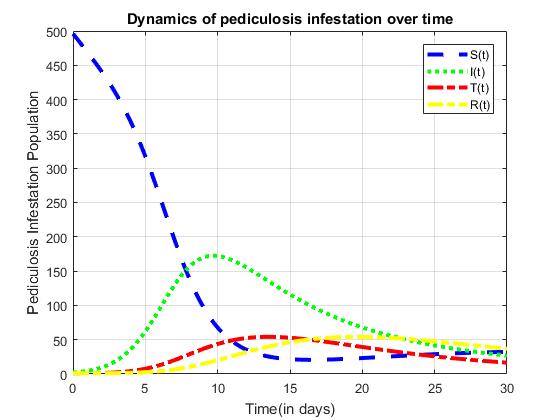
**Table 3**. Parameter used in model simulations

|  |  |  |
| --- | --- | --- |
| **Parameter Values** | **Source** |  |
| 0.05 | [17] |  |
|  |  |  |
| 0.002 | [8] |  |
|  |  |  |
| 0.10 | [17] |  |
|  |  |  |

 0.10 Assumed

 0.20 Assumed

 0.05 [17]

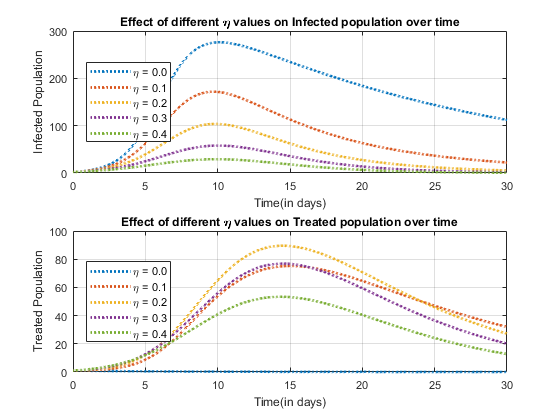
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**Fig. 2** Dynamics of Pediculosis infestation over time for different parameter values.

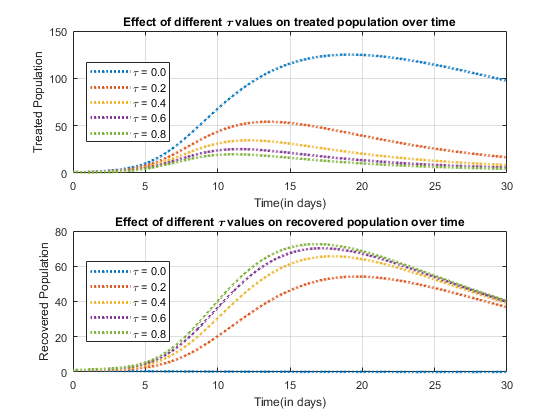
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|  |

**Fig.3** Effect of different  values on susceptible and infected population over time.

|  |
| --- |
|  |

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**Fig.4** Effect of different values on infected and treated population over time

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|  |
| --- |
|  |
|  |

**Fig.5** Effect of different values on infected and treated population over time.

1. **Optimal Control Problem**

In this section, our interest is to curtail the spread or transfer of pediculosis and to achieve this we need control. We extend the system of model equation (2.1) - (2.4) by incorporating two optimal control and The first control correspond to decrease in the rate of transmission () and the second control correspond to the increase in the treatment rate ().

Our main goal of incorporating these controls is to minimize the rate of transmission between individuals who are susceptible and individuals who are infectious and also to maximize the treatment rate with minimal control cost. The two controls (education) and (treatment) are both bounded and lebesgue integrable on the interval where is the initial time and  is the final time. In other to have maximum control effort, the two controls and must be equal to one.

Thus, the optimal control system of model equation is as follows



we define our objective functional as follows



Where the constants are the weight constants. The weight constant is the relative measure of the significant of decreasing the infectious individuals while and are the weight constants related to education and treatment cost respectively.

Our goal is to determine the optimal control and  such that 

Where U is the set of controls and is defined as



We defined the Langragian of this problem as



Our choice of the coefficient ½ attached to  and in both the objective functional and langragian is for ease in computation.

**Theorem 3.**

Let the optimal control corresponding to the state variables  be there exist adjoint variables which satisfy





Equation (3.7) is the transversality conditions.

With the optimal controls and  defined as



Where



**Proof**

In other to prove the theorem above, we use the Pontryagin’s maximum principle [22]. Based on the optimal control system of model equation (3.1) – (3.4) and the objective functional in (3.5), we formulate the Hamiltonian functional H as follows



Where are co-state or adjoint variables.

We obtain equation (3.6) from



Since all the costate are free at the final time, the transversality condition is



To obtain the optimal control we differentiate the Hamiltonian function with respect to and on and equate to zero.

Thus,



Solving equation (3.8) for  and  yields



Hence we have



And



Expressing the optimal control and  in their simplest form, yields



Thus, this completes the proof of the theorem above.

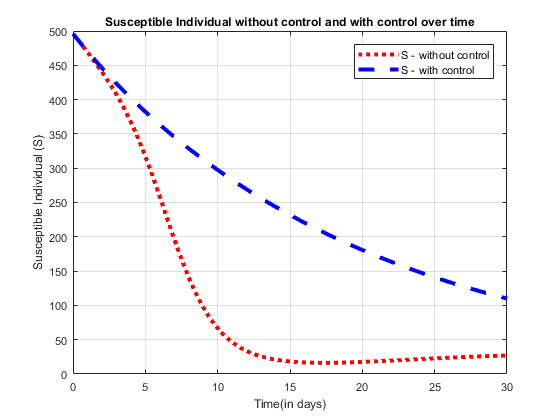
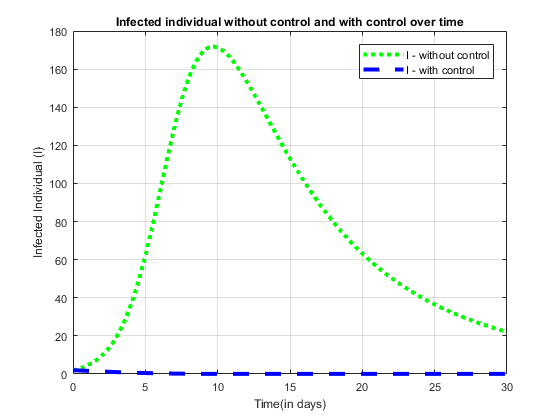
Therefore, our optimality system to be solved is



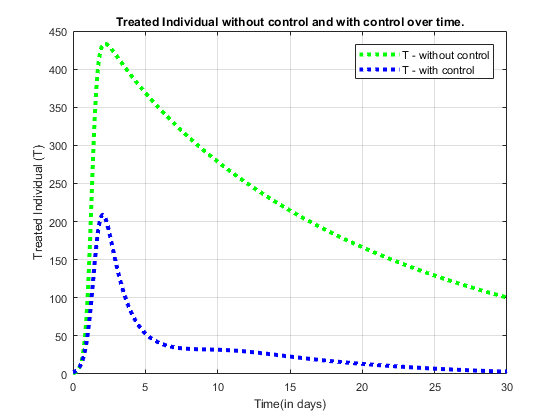
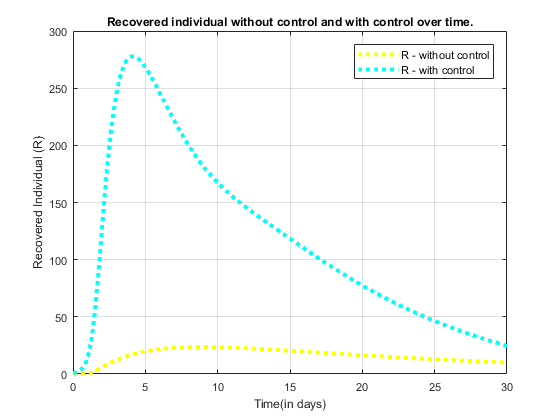
**3.1 Numerical Simulation of the Optimal Control Model**

We computed numerically the resulting optimal control problem by using a forward-backward sweep method. To achieve this, we solve the model state equation using a forward fourth-order Runge-Kutta scheme with an initial guess for the controls and in time interval  while we use backward fourth-order Runge-Kutta scheme to solve the costate or adjoint equation. We use the parameters in table 2 with the control the initial condition  and the constant weight  to iteratively compute.

Lastly, we assume two maximum effective values for the control strategies and to simulate the optimal control problem.

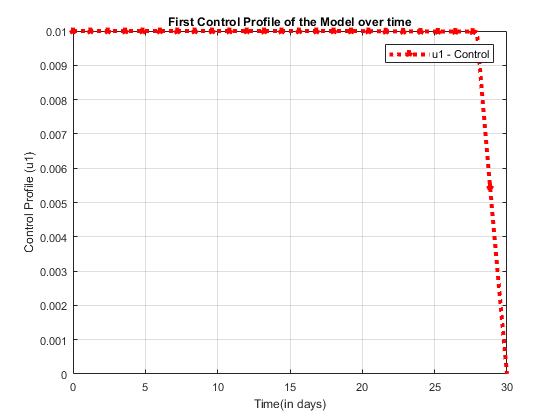
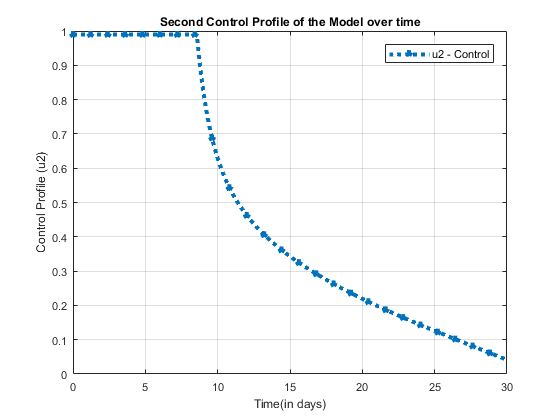
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**(a) Susceptible Individual**  **(b) Infected Individual**

** **

**(c) Treated Individual**  **(d) Recovered Individual**

**Fig.6 Comparison between population without control and population with control over time**.

** **

**(e) Optimal control u1** **(f) Optimal control u2**

**Fig.7 Different optimal control model profile over time.**

**Discussion of Result**

First, in figure (6a), the population of susceptible individuals are shown without control and with control (education and treatment). We observe that, it takes shorter time for population of susceptible individuals without control to decrease continuously nearly to zero unlike the population of susceptible individual with control which take longer time decrease. This implies that, application of control helps to reduce the chances of children from being infected.

Also, in figure (6b), population of infected individuals are displayed without control and with control. We noticed that, in the absence of control, the population of individuals who are infected increase continuously and takes longer time to reduce, however it completely reduce to zero in the presence of control. This entails that, when control is applied it helps to attenuate infection rate from children who are infected with head lice. Thus we recommend these controls to schools with respect to head lice spread mitigation among children.

In addition, we see in figure (6c) that population of individuals who are treated of head lice are higher without control and takes longer time to reduce to zero compared to population of individuals who are treated with controls and takes shorter time to reduce. This suggest that in absence of control, the number of infected individuals would rise, thus resulting to increase in the number of children to be treated.

Furthermore, in figure (6d), it is obvious that population of individuals who recovered from head lice with control increase continuously and takes longer time to reduce to zero, which is in contrast to population of individuals who recovered from the disease without control. It infers that when control is applied, individuals who receive education and treatment would recover faster compared to individuals who didn’t embrace education and treatment.

Lastly, in figure (6e) and figure (6f), the control profile of the model are presented. The control in figure (6e) represent education while that of figure (6f) represent treatment. We observe in figure (6e) that continuous education is required to reduce head lice spread amongst the population of children. In figure (6f), we observe that the corresponding optimal control (treatment) is to be applied at the largest admissible dose up to eight (8) days and then reduce it strength as the population of individuals with head lice keeps reducing.

**Conclusion**

This paper present and analysed an epidemic model for transmission dynamics of head lice spread. The model consists of state variables expressed in form of nonlinear ordinary differential equation. The proposed model has in existence one equilibrium (disease free equilibrium) which is locally asymptotically stable provided basic reproduction number  is less than one  Lastly, we extended the proposed model to have an optimal control system by incorporating two controls (education and treatment). We analysed the optimal control system, solved it numerically using forward-backward sweep fourth order Runge-Kutta method and carried out numerical simulation in Octave/MATLAB to investigate the effect of the controls on the state variables. It was observed that results with optimal controls are far better than results without optimal controls. Thus, we strongly recommend these controls (education and treatment) to attenuate head lice spread amongst school children.

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**Author’s Biography**

A person with a mustache and hand on chin

AI-generated content may be incorrect.**Emeka Emmanuel Otti** is a mathematician with a strong academic background and diverse research interests. He is a Lecturer and researcher in Mathematics and Statistics Department, Federal University Wukari, Nigeria. He obtained his B.Sc. in Mathematics from Federal University Wukari, Taraba State, Nigeria, in 2016, and later earned an M.Sc. in Mathematics from the African Institute for Mathematical Sciences (AIMS), Senegal, in 2020. Currently, he is pursuing his doctorate degree in Applied Mathematics at Case Western Reserve University, USA. His research focuses on Mathematical Modeling, Biomathematics and Optimization, where he uses survival model as a mathematical tool to understand and predict outbreaks and help to estimate how long people remain infectious.